Private Equity Fund Returns: Do Managers Actually Leave Money on the Table?*

Robert Marquez
Boston University

Vikram Nanda
Georgia Tech

M. Deniz Yavuz
Washington University in St. Louis

June 1, 2010

*Contact: Deniz Yavuz (deniz.yavuz@wustl.edu). We would like to thank Philip H. Dybvig, Alex Edmans, Paolo Fulghieri, Thomas F. Hellman, Steven N. Kaplan, Laura Lindsey, Umit Ozmel and seminar participants at Arizona State University and the 6th Annual Corporate Finance Conference, 2009.

Private Equity Fund Returns: Do Managers Actually Leave Money on the Table?

Abstract

Evidence indicates that private equity funds, unlike mutual funds, deliver persistent abnormal returns and that the top performing funds are often oversubscribed. Why do private equity funds appear to leave money on the table, rather than increasing fund size? We argue that private equity funds are fundamentally different from mutual funds because their success is contingent on their matching with high quality firms (entrepreneurs). Firms also want to match with managers that have higher estimated value adding ability. This gives managers an incentive to manipulate firms' beliefs about their ability by providing higher returns. Managers limit fund size, fees or both even if firms are not fooled in equilibrium. The model explains the puzzle and provides several new time series and cross sectional predictions about fund performance, fees and size. For example, venture capital funds should have stronger performance persistence and lower sensitivity of size to past returns compared to that of buyout funds.

Keywords: venture capital, private equity, performance persistence, signal jamming, leveraged buyout, fund size and fees.

1 Introduction

Anecdotal evidence abounds that some successful private equity funds do not accept all the money that investors are willing to invest.[1] At the same time, these funds appear to generate persistent abnormal returns for their investors (see, e.g., Kaplan and Schoar, 2005; Phalippou and Gottschalg, 2008).[2] Kaplan and Schoar (2005) conjecture that top-performing funds voluntarily restrict their size and that their performance is in striking contrast to the performance of successful mutual funds that exhibit no performance persistence (Berk and Green, 2004).[3] This raises an obvious question: Why do successful private equity funds not capture higher rents by increasing fund size or fees?

To address this puzzle we focus on one of the fundamental differences between private equity and mutual funds. Unlike mutual funds, many private equity funds, such as venture capital funds, not only have to identify good investment opportunities but they also need to persuade entrepreneurs (firms) to accept their offers. On their part, firms also want to match with private equity funds that are more likely to add value. Private equity funds may add value through providing strategic advice, helping to professionalize firms' management, and attracting better resources, business partners and human capital.[4] Hsu (2004) provides evidence that firms are both more likely to accept offers from and sell their shares at a discount to VCs that are more reputable and have a higher ability to add value. This positive assortative matching process, where funds with high ability to add value match with good firms, is likely to be very important for the success of the private equity funds.[5] Indeed, Sorensen (2007) provides evidence that VC fund success can be largely explained by

[1]Several cases in which VC funds were oversubscribed are noted in the article *Oversubscribed,* European Venture Capital Journal, November 2006. Also see Kaplan and Schoar (2005).

[2]See also Quigley and Woodward (2002), Jones and Rhodes-Kropf (2003), Ljungqvist and Richardson (2003), Cochrane (2005), and Korteweg and Sorensen (2008), which explore the risk, cash flow and performance of private equity funds. Phalippou (2009) shows that the finding of performance persistence is stronger when investors are unsophisticated but disappears when they are sophisticated.

[3]The worst performing mutual funds do exhibit some performance persistence (see Carhart, 1997), possibly resulting from inattention by investors in these funds.

[4]See Gorman and Sahlman (1989), Megginson and Weiss (1991), Hellmann and Puri (2000, 2002), Baum and Silverman (2004).

[5]Matching is important also in financial intermediation (Chemmanur and Fulghieri, 1994; Fernando, Gatchev and Spindt, 2005; Bao and Edmans, 2009).

experienced VCs matching with better firms.

Although firms want to match with the best funds, they do not directly observe a fund manager's ability to add value and have to infer it from the fund's past performance. Private equity funds, unlike mutual funds, are largely exempt from public disclosure requirements (Kaplan and Schoar, 2005). Information about past returns is available, but is generally self-reported either by general or limited partners. Even if past return data is available, it is difficult to estimate a manager's ability to add value because generally sufficient information is not available about the quality of the firms in the manager's portfolio, the exact fees charged and the fund's size, especially if the fund has not fully exited from all of its investments (Phalippou and Gottschalg, 2008; Cumming and Walz, 2009).

We argue that this matching process together with lack of information to estimate the ability of managers can explain the difference in return patterns between mutual and private equity funds. Managers may have an incentive to manipulate firms' beliefs about their ability to add value, which has important implications for fund performance, size and fees.

We present a simple model that incorporates these features. As a baseline, we first study the case where there are no information asymmetries, so that firms observe all factors that can affect their estimation of the manager's ability. In this case our model collapses to a version of the model in Berk and Green (2004). The manager cannot manipulate the beliefs of firms because they can easily estimate the manager's ability to add value. Here, we find that the manager accepts all the money offered by investors and captures all the expected surplus.

We then analyze the more realistic case where firms are not perfectly informed about various aspects of the manager's actions or fund characteristics that can affect fund returns. We first assume that fees are observed and set at industry benchmarks (we relax this later). We first introduce unobserved managerial effort and show that the manager exerts higher effort than when effort is observable. He also limits the size of the fund by accepting a lower quantity than what investors are willing to offer. Limiting fund size increases the marginal effect of effort on fund returns. However, firms conjecture the manager's actions and are not fooled in equilibrium. Regardless, the manager cannot avoid the attempt to manipulate firms' beliefs because firms cannot differentiate between managers that do not exert excess effort from those who have lower ability to add value.[6] As a result the fund manager limits his fund's size and provides a positive and persistent expected return (alpha) to investors.

Although it is natural to assume that managerial effort is unobservable, our results do not depend on this assumption. We consider several cases where there is no managerial effort but where the fund manager can affect returns through fund characteristics that are not fully observed by firms. We first introduce information asymmetry regarding fund fees and let the manager select the fees charged by the fund. Most funds use a 1.5-3 percent management fee and 20-25 percent carry interest (Gompers and Lerner 1999; Phalippou and Gottschalg 2008; Litvak 2009). However, estimating actual fees charged can be complicated in practice. For example, distributions may generate an interest-free loan to VCs from limited partners, which Litvak (2009) argues might be as important as the management fee or carry interest. In this case, we show that the manager not only charges lower fees compared to the benchmark case, but also limits the fund size. The fund's expected alpha is again positive and persistent.

We next introduce information asymmetry regarding the size of the fund. Total committed capital is sometimes observable by outsiders. However, it may not be possible to observe the actual quantity at the time a subsequent fund is established because the manager can give back funds to investors if he cannot find good investment opportunities. In our model, information asymmetry regarding fund size could also be considered as a proxy for information asymmetry regarding dis-economies of scale (Lopez-de-Silanes, Phalippou and Gottschalg, 2009; Cumming and Dai, 2010). We show that the manager limits the fund's size, which serves to provide positive and persistent expected alpha to fund investors.

We also consider a number of interesting combinations of these cases. For example, in the special case when the manager is allowed to select fees and these are fully observable, unobservability of size or effort still causes the manager to limit the size of the fund. However,

[6] This is a classical "signal jamming" equilibrium (Holmstrom, 1999; Stein, 1988), in which an agent tries to affect the principal's perception of his ability by manipulating the signal.

the manager captures all the surplus from investors because adjusting the fees does not change firms' beliefs about his ability. Even in this case it is possible to obtain performance persistence in a variation of the model in which managers use the fee structure to commit to keep the fund's size small. This may occur because entrepreneurs have some outside option that then only makes it worthwhile to accept a manager's offer when his fund is relatively small. Given that fund size is a function of fees, managers may use fees to pre-commit to a certain size. Again, the manager provides positive alpha to investors only when there is an incentive to manipulate beliefs of investors, thus reinforcing our previous results.

In all cases the manager voluntarily limits fund size, as conjectured by Kaplan and Schoar (2005), in order to improve his ability to match with good firms in the future. In addition to explaining the performance persistence puzzle, we provide several cross sectional and time series predictions about the performance of private equity funds. Our model explains the observed differences in performance persistence between venture capital and buyout firms. Empirically, it is known that buyout firms scale their size much faster in reaction to past positive returns (Metrick and Yasuda, 2008) and that their performance persistence is lower (Kaplan and Schoar, 2005) compared to venture capital funds. This is consistent with our findings, since if target firm managers (entrepreneurs) do not have a continuing interest in the firm they only care about who offers the best buyout price and have no particular preference for matching with high ability fund managers. In most buyout deals existing shareholders sell the entire company, often for cash, which removes their incentive to care about the fund managers ability to add value. As a result we expect our arguments to be more applicable to venture capital funds than buyout funds. This distinction between the different kinds of funds is, to the best of our knowledge, unique to our framework.

There are also cross sectional differences among venture capital funds in terms of their focus on early versus later investment stage (Gompers and Lerner, 1999). The matching process should be more important for venture capital funds that invest in early stage companies given that the VC's added value is more pronounced in the first two rounds of financing (Chemmanur, Krishnan and Nandy, 2009). To the best of our knowledge whether performance persistence changes with the stage of focus of venture capital funds has not been tested yet.

Although we do not explicitly model how competition affects matching process, managers' incentives to manipulate firms' beliefs should depend on the degree of competition among venture capital funds to attract good target firms in a nonliner fashion. With no competition, a manager has no incentive to manipulate beliefs. On the other hand, too much competition will decrease the marginal impact of manipulation on the matching probability with good firms. Therefore, we expect most of the effect to be present for intermediate levels of competition. To the extent that individual markets may exhibit different degrees of competition - either because of geographic separation (Chen, Gompers, Kovner and Lerner, 2009), specialization (Fulghieri and Sevilir, 2009; Gompers, Kovner, Lerner, Scharfstein, 2008) or over time - this presents an as yet untested prediction concerning how managers' incentives and performance persistence may vary in the cross-section and time series.

In the time series, as the manager gets more experienced, firms have better information about the ability of the manager. Therefore, over time a fund manager's incentive to manipulate firms' beliefs should decrease. As a result we expect a positive relationship between past fund returns and changes in fund size, consistent with the evidence in Kaplan and Schoar (2005). Our model also predicts that the learning that takes place over time about a fund manager's ability implies larger fees by more experienced managers (see, e.g., Gompers and Lerner, 1999).

Glode and Green (2008) and Hochberg, Ljungqvist and Vissing-Jorgensen (2008) provide alternative explanations based on the observation that private equity fund investors are mostly institutions as compared to individuals in mutual funds. In Glode and Green (2008) investors learn about the fund's investment strategy and they can potentially share this information with other non-incumbent managers. In equilibrium, funds limit their size and deliver excess returns to investors, thereby inducing them to not divulge information about the fund's strategy. By contrast, Hochberg, Ljungqvist and Vissing-Jorgensen (2008) argue that past fund investors have private information concerning the fund manager's skill.

Investors extract rents by threatening to hold up the fund manager when he next starts a fund because other potential investors interpret failure to reinvest by incumbent investors as a negative signal about the manager's skill. However, Busse, Goyal and Wahal (2009) do not find performance persistence in mutual funds that cater to institutional investors and Phalippou (2009) finds that venture capital funds that are expected to be backed by more skilled investors show no performance persistence. These indicate that other fundamental differences between private equity and mutual funds could also be at play. Our work complements these papers by providing an asset-side explanation for why private equity and mutual funds exhibit different performance persistence.

Several recent papers analyze the optimal size of venture capital firms. For example, Fulghieri and Sevilir (2009) show that a venture capitalist may limit fund size when it is important to provide entrepreneurial incentives to firms. A small portfolio increases the value-added to each firm by the VC and encourages entrepreneurs to exert higher effort, while a large portfolio allows the VC to reallocate resources in the case of startup failure and to extract higher surplus from entrepreneurs. Inderst, Mueller and Muennich (2007) argue that limiting the size benefits the venture capitalist by decreasing the bargaining position of portfolio firms and introducing higher competition for available funds. Our results indicate that a venture capitalist may also limit size in order to improve the fund's probability of matching with good firms in the future when there is information asymmetry regarding managers' ability to add value.

2 Model

In any period t, there is a continuum of firms, which can either be "good" or "bad." Good firms are defined as those for whom a fund manager can add value. Bad firms are those for which the fund's investment adds no value. These firms need to raise a fixed amount of financing, normalized to \$1, in return for giving a fraction of the company to the investing fund. Given that the initial owners of the firms (e.g., entrepreneurs) do not sell all the shares of the company, both the firms' owners as well as the investing fund managers benefit from any future increase in the firms' value. The quantity of shares and the price at which a firm's shares are sold to a fund affects the manner in which any value created is shared between the fund and the entrepreneur. We assume that both parties receive a strictly positive share of the value created.[7]

The return from investing in good firms depends on the manager's ability to add value. A fund manager's innate ability or talent is given by X, which is distributed according to $N(\overline{X}, \sigma_X^2)$. The added value delivered by a fund manager to a firm in which the fund invests in any period is $X + \epsilon_t$ percent, where ϵ_t is a random shock in that period. The shock ϵ_t is i.i.d. over time, and is distributed as $N(0, \sigma_\epsilon^2)$. Neither the manager nor the target (i.e., portfolio) firms know the actual realization of X. Each firm would like to match with the manager who has the highest expected value added $E[X]$ among the offers the firm receives. Good firms are more likely to get multiple offers, so that a firm's beliefs about the fund manager's ability to add value is critical for the fund to match with good firms. We assume that the fraction of money in the fund manager's portfolio that is invested in good firms at time t, P_t, increases with the expected value added of the manager, $E_t[X]$. Therefore, matching between firms and managers is positive assortative; that is, managers with higher perceived ability are more likely to match with good firms. Positive assortative matching is a common prediction of various models (Titman and Trueman, 1986, Chemmanur and Fulghieri, 1994, Fernando, Gatchev and Spindt, 2005), and we take it as given here.

Following Berk and Green (2004), we assume that the per unit cost of adding value to good firms is $c(Q_t)$, which is independent of ability and is increasing and convex in the amount of funds under management at time t, Q_t. This assumption therefore implies decreasing returns to scale in private equity, as documented by Lopez-de-Silanes, Phalippou and Gottschalg (2009). The gross percentage return of the fund is equal to $W_t = X + \epsilon_t -$

[7] In the extreme case, the fund manager may be able to capture all the surplus he will generate for the firm ex-ante by paying a lower price for the shares acquired. However, if both parties are essential for this transaction, it is reasonable to assume that both parties will share the surplus in some way, such as through Nash bargaining. If the fund manager captures all the value he generates, the firms will be indifferent to the manager's talent.

$c(Q_t)$.

The fund manager raises capital from investors and charges a fixed fee f_t, which is a percentage of the committed funds Q_t, as well as a variable fee $v_t \leq 1$, which is a percentage of the fund's return net of the fixed fee There is symmetric information between investors and the fund manager, i.e., investors observe the manager's choice of fees and quantity. Investors are competitive and hence they are willing to provide capital as long as their net return, α_t, is nonnegative. Given the total return $P_t W_t$ to the fund in period t, the net return to investors can be written as $\alpha_t = (1 - v_t)(P_t W_t - f_t)$. Firms, by contrast, observe only the fund's history h_t, which includes all realized α_i for $i < t$ and includes information about fund characteristics such as size or fees depending on our assumptions about observability.[8] We first incorporate unobservable managerial effort that improves fund returns. Later, we consider a variety of different cases depending on whether the fees (f_t, v_t) and/or the fund's size (Q_t) are observable by the target firms.

The manager maximizes his payoff by choosing a fee structure and quantity to invest considering the participation constraint of the investors and the impact of his actions on firms' beliefs about his ability to add value. We can write the total future payoff for the fund manager as of time t as

$$V_t = \sum_{i=t}^{\infty} \delta_t \left[Q_i v_i (P_i W_i - f_i) + Q_i f_i \right],$$

where δ_t is the discount rate, which we ignore in the rest of the paper for brevity.

2.1 The Case of no Information Asymmetry

We begin with a benchmark scenario where there is no information asymmetry between the fund manager and firms. Specifically, firms perfectly observe the manager's choice of

[8] Note that when $v_t = 1$, $\alpha_t = 0$ and investors cannot calculate the gross return of the fund. This special case does not affect any of our arguments below because the fund manager can also capture all the surplus by charging a fixed fee that is equal to the gross returns: $f_t = P_t W_t$. Therefore, in cases when the manager captures all the surplus we simply assume that he does so through the fixed fee, which makes the variable fee redundant.

quantity Q_t and fees v_t and f_t. Firms can then use the information in the history of the fund, which includes fees, fund size, and past returns, to estimate the manager's ability to add value, X. Bayesian updating after observing α_t gives us the conditional expectation of X at time $t+1$ as:

$$E_{t+1}\left[X|h_t\right] = w_t E_t[X|h_{t-1}] + (1-w_t)\left(\frac{\alpha_t}{(1-v_t)P_t} + \frac{f_t}{P_t} + c\left(Q_t\right)\right),$$

since $\alpha_t = (1-v_t)(P_t W_t - f_t)$ and $W_t = X + \epsilon_t - c\left(Q_t\right)$. This simplifies to

$$E_{t+1}\left[X|h_t\right] = w_t E_t[X|h_{t-1}] + (1-w_t)\left(X + \varepsilon_t\right).$$

The weight w_t depends on the relative precision of new information and previous information about X. Note that firms' expectations about the manager's ability at time $t+1$ are not affected by the manager's decision variables Q_t, f_t or v_t:

$$\frac{\partial}{\partial Q_t} E_{t+1}\left[X|h_t\right] = 0, \quad \frac{\partial}{\partial f_t} E_{t+1}\left[X|h_t\right] = 0, \frac{\partial}{\partial v_t} E_{t+1}\left[X|h_t\right] = 0.$$

Since the manager's policy decisions today do not affect firms' expectations, they also do not affect his payoffs in the future. As a result, the manager chooses the fee structure and the size of the fund to maximize his payoff subject to the constraint that investors require a nonnegative return. We can write the Lagrangean of the optimization problem as

$$\max_{Q_t, f_t, v_t} \sum_{i=t}^{\infty} Q_i v_i (P_i W_i - f_i) + Q_i f_i + \lambda_t (1-v_t)(P_t W_t - f_t),$$

where λ_t is the Lagrange multiplier on the investors' participation constraint. We can now state the following result, which characterizes the solution to the fund manager's problem.

Proposition 1 *The manager's profit is maximized at a* Q_t *such that* $-Q_t c'(Q_t) + X_t - c(Q_t) = 0$. *The equilibrium profit at time* t *of the fund manager is equal to:* $\Pi_t = Q_t[X_t -$

$c(Q_t)]$.

Note that the manager's choice of Q_t is independent of the fee structure and is chosen simply to maximize the total value generated. As the next result shows, the manager then chooses the fees v_t and f_t to capture all the value created.

Proposition 2 *The manager choses v_t and f_t such that fund investors' expected excess return α_t is zero.*

When there is no information asymmetry between the manager and firms in estimating the manager's ability, the manager has no way of manipulating the beliefs of firms. As a result, the manager maximizes his return by extracting all the surplus from investors through the fee structure.

Next we extend the base model by introducing effort. We introduce costly managerial effort that improves the fund's returns by mitigating the cost of allocating funds. In other words, the manager's effort affects fund returns but not the value added to firms, which is still determined purely by the manager's talent. We assume that the cost of effort, for the same level of effort, is the same for all managers. We choose this particular structure to ensure that firms try to infer managers' talent X, but are indifferent as to the equilibrium level of effort or cost of effort.

The gross return delivered by a fund manager in any period t is now given by $W_t = X + e_t + \epsilon_t - c(Q_t)$, where e_t denotes the manager's effort. Effort is privately costly to the manager. The cost of effort $QC(e_t)$ is increasing and convex in effort, and is increasing with the size of the fund because the marginal effect of effort on fund returns should be lower for larger funds.

Again to establish a benchmark, we assume for now that firms observe managerial effort, which ensures that the manager's effort choice has no effect on firms' expectations. The Lagrangean of the optimization problem is:

$$\max_{Q_t, f_t, v_t, e_t} \sum_{i=t}^{\infty} Q_i v_i (P_i W_i - f_i) + Q_i f_i - Q_i C(e_i) + \lambda_t (1 - v_t)(P_t W_t - f_t).$$

When there is no information asymmetry about the effort and other choice variables, the manager maximizes the surplus he generates for the current period and captures all this surplus.

Proposition 3 *The manager choses e_t, Q_t , v_t and f_t such that investors' expected excess return is zero.*

2.2 Asymmetric Information

While in the benchmark cases above we assumed symmetric information between firms and the fund manager, it is likely that certain characteristics of the fund that directly affect realized returns to fund investors are not observed by outsiders (see, e.g., Phalippou and Gottschalg, 2008, or Cumming and Walz, 2009). Information asymmetry about any variable the manager can control and which affects gross returns can prevent firms from perfectly deducing the manager's talent. These may include the quality of the firms in fund's portfolio, the risk of the portfolio, the exact fees charged, the fund size, and managerial effort, among other. The problem of inferring the value-adding ability of the manager is even more severe for funds that have not fully exited all of their investments. We next analyze the case where some information necessary to estimate a manager's ability is not observed by the portfolio firms.

2.2.1 Unobserved Effort

One obvious source of information asymmetry between firms and the manager is managerial effort. To isolate the effect of unobservable effort, here we assume that both the quantity under investment and all fees are perfectly observed by the firms and that fees are set at some fixed level $v_t < 1$ and $f_t < P_t W_t$; later we allow the manager to choose the fees. Define the conjectured value of effort as e^C. Expectations about the manager's ability depend on the history of realization of net returns for investors, and are given by

$$E_{t+1}\left[X|h_t\right] = w_t E_t[X|h_{t-1}] + (1-w_t)(X + \epsilon_t + e_t - e_t^C).$$

The manager's effort choice clearly affects the perceived talent next period since

$$\frac{\partial}{\partial e_t} E_{t+1}\left[X|h_t\right] > 0.$$

Given that the manager's actions affect the expectations of managerial talent in the future, the maximization problem of the manager now becomes a dynamic problem. In this dynamic optimization problem the state variable is firms' beliefs about the ability of the manager. The choice variables are effort and fund size. We can write the Bellman equation as follows:

$$V_t(E_t\left[X|h_{t-1}\right]) = \max_{Q_t, e_t} \left[Q_t v_t(P_t W_t - f_t) + Q_t f_t + V_{t+1}(E_{t+1}\left[X|h_t\right])\right]$$

subject to the constraints

$$(1-v_t)(P_t W_t - f_t) \geq 0,$$

$$E_{t+1}\left[X|h_t\right] = w_t E_t[X|h_{t-1}] + (1-w_t)(X + \epsilon_t + e_t - e_t^C).$$

The first constraint ensures that investors are willing to participate and the second shows how the state variable changes over time and provides the link between the value functions V_t. Given that the second constraint is an equality, we can plug in the expected value $E_{t+1}\left[X|h_t\right]$ in V_{t+1} and write the Kuhn-Tucker conditions as follows:

$$v_t(P_t W_t - f_t) - Q_t v_t P_t c'(Q_t) + f_t - \lambda_t P_t c'(Q_t)(1-v_t) + C(e) + \frac{\partial V_{t+1}}{\partial E_{t+1}[X]} \frac{\partial E_{t+1}[X]}{\partial Q_t} = 0 \quad (1)$$

$$\lambda_t(1-v_t)(P_t W_t - f_t) = 0 \quad (2)$$

$$Q_t v_t P_t - Q_t \frac{\partial C(e_t)}{\partial e_t} + \frac{\partial V_{t+1}}{\partial e_t} = 0 \tag{3}$$

The solution to the problem indicates that the fund manager exerts higher effort compared to the case when effort is observable. Firms, in turn, recognize the manager's incentive to mislead them and update their conjectures accordingly. In equilibrium, firms' conjectures will be correct and the manager will not fool the firms. Nevertheless, the manager will attempt to manipulate firms' beliefs by increasing his effort because otherwise he would be perceived as having low ability. In addition, the manager accepts a lower quantity of funds than what is optimal when effort is observed despite the fact that his choice of quantity is observed by the firms. This occurs because managing a larger quantity makes it more costly to provide the same expected return to investors as a way of manipulating the beliefs of firms. By decreasing quantity, the manager exerts higher effort and hopes to have a higher impact on his probability of matching with good firms in the future.

Proposition 4 *When effort is unobserved, the manager both exerts higher effort and limits fund size compared to when effort is observed. Fund investors' excess return,* α_t*, is positive and persistent.*

Although unobservable effort is a reasonable assumption, our main results do not depend on this assumption. Several choice variables of the manager that affect reported returns but are not observable by firms provide similar incentives to the manager to manipulate firms' beliefs. Next we remove effort and consider unobserved fund characteristics, such as fees or fund size.

2.2.2 Unobserved Fees

In the previous section we assumed that fees are common knowledge and set at some exogenously fixed level. Now we consider the more realistic case where fees are not perfectly observed by outsiders and the manager can choose the level of fees. Although most funds use a 1.5-3 percent management fee and a 20-25 percent carry interest (Gompers and Lerner, 1999; Phalippou and Gottschalg, 2008; Litvak, 2009), it is reasonable to assume that there is some uncertainty regarding exact fees paid and that the manager has some discretion in determining the level of these fees. For instance, there may be hidden fees such as distributions that generate an interest free loan to VCs from limited partners (Litvak, 2009) or kickbacks that rewards investors. To isolate the effect of unobservability of fees, we assume that fund size is observed by the firms.

In this case, firms conjecture the fee structure f_t^C and v_t^C and use the fund's past history to estimate the manager's talent:

$$E_{t+1}\left[X|h_t\right] = w_t E_t[X|h_{t-1}] + (1-w_t)\left(\frac{\alpha_t}{(1-v_t^C)P_t} + \frac{f_t^C}{P_t} + c\left(Q_t\right)\right),$$

which is equal to

$$E_{t+1}\left[X|h_t\right] = w_t E_t[X|h_{t-1}] + (1-w_t)\left(\frac{(1-v_t)(P_t(X+\epsilon_t - c\left(Q_t\right)) - f_t)}{(1-v_t^C)P_t} + \frac{f_t^C}{P_t} + c\left(Q_t\right)\right).$$

The manager's choice of fees today clearly affects the talent perceived by firms next period. If the manager charges a lower fixed or variable fee than conjectured by the investors, he can hope to convince investors that he has superior talent, which will increase the fraction of good firms in the manager's portfolio in the future. This effect can be seen clearly from the derivative of the conditional expectation of talent with respect to the manager's choice variables,

$$\frac{\partial}{\partial f_t}E_{t+1}\left[X|h_t\right] < 0, \frac{\partial}{\partial v_t}E_{t+1}\left[X|h_t\right] < 0.$$

We can now write the Bellman equation for the manager:

$$V_t(E_t\left[X|h_{t-1}\right]) = \max_{Q_t, f_t, v_t}\left[Q_t v_t(P_t W_t - f_t) + Q_t f_t + V_{t+1}(E_{t+1}\left[X|h_t\right])\right]$$

subject to the constraints

$$(1-v_t)(P_t W_t - f_t) \geq 0$$

$$E_{t+1}\left[X|h_t\right] = w_t E_t[X|h_{t-1}] + (1-w_t)\left(\frac{\alpha_t}{(1-v_t^C)P_t} + \frac{f_t^C}{P_t} + c\left(Q_t\right)\right)$$

When fees are not observed, firms cannot perfectly infer the manager's ability from the fund's past returns. The manager therefore has an incentive at the margin to reduce his fees so as to increase the return to investors with the hope of improving firms' perceptions of his ability. In this case, the manager provides positive expected return α_t to the fund's investors and does not capture all the surplus. Given that the fees are lower, investors are actually willing to invest more than in the benchmark case. However, the manager limits the size of the fund and does not accept all funds offered by investors. As a result, the fund's size could be larger or smaller than in the benchmark case.

Proposition 5 *When fund fees are not perfectly observed by firms, the manager limits fees and fund size. Fund investors' expected return α_t is positive and persistent.*

In equilibrium, of course, firms correctly conjecture the manager's incentives and anticipate the actual fee structure that will be chosen. That is, the manager is again unable to fool the market, and his ability is inferred in a unbiased way. Nevertheless, the asymmetry of information leads the manager to leave money to investors through his incentives to attempt to manipulate firms' beliefs.

2.2.3 Unobserved Fund Size

We next consider the case where firms are unsure about the actual quantity that will be placed by the manager, so that the overall fund size is not observed perfectly. That actual fund size may not be fully observed, especially when a manager establishes a new fund before fully exiting the previous one, is realistic. Moreover, in our model fund size is also used to capture dis-economies of scale that could be the result of various factors including the workload of managers (Lopez-de-Silanes, Phalippou and Gottschalg, 2009). Therefore, one could also consider this case as relating to an information asymmetry about the workload of the manager or dis-economies of scale in general. We again assume that fees are set at some fixed level $v_t < 1$ and $f_t < P_t W_t$.

The structure of the problem remains the same as before but this time firms conjecture the size of the fund Q_t^C rather than the fees in trying to estimate the talent of the manager. Firms update their beliefs as follows:

$$E_{t+1}\left[X|h_t\right] = w_t E_t[X|h_{t-1}] + (1-w_t)\left(X + \epsilon_t - c(Q_t) + c(Q_t^C)\right) \tag{4}$$

From (4), it is clear that decreasing fund size positively affects firms' estimation of talent. As a result, the manager limits the fund's size and provides positive and persistent returns to fund investors.

Proposition 6 *When fund size is not observed, the manager limits the fund's size. Fund investors' expected return* α_t *is positive and persistent.*

2.2.4 Combinations of Unobservability of Size and Fees

In this section we study two interesting cases. In the first case, firms observe neither the fees nor the quantity choices of the managers. In the second case, firms do not observe quantity but they do observe fees, which can be freely determined by the manager.

In the first case firms' expectations of the manager's ability are given by:

$$E_{t+1}\left[X|h_t\right] = w_t E_t[X|h_{t-1}] + (1-w_t)\left(\frac{(1-v_t)(P_t(X+\epsilon_t - c\left(Q_t\right)) - f_t)}{(1-v_t^C)P_t} + \frac{f_t^C}{P_t} + c\left(Q_t^C\right)\right).$$

The manager's choices clearly affect the talent perceived by firms next period. If the manager charges a lower fixed or variable fee and invests a lower amount than conjectured by the investors, the manager can hope to convince investors that he has greater talent. Therefore it is not surprising that we obtain similar results as before.

Proposition 7 *When both fund size and fees are unobserved, the manager limits fund size and fees. Fund investors' expected return* α_t *is positive and persistent.*

In previous sections we assumed either that fees are set at some fixed level or that they are not fully observed. We next analyze the special case when fees can be freely chosen by the manager, but are perfectly observed by firms. Fund size, however, is not observable. One may argue that when fees are observed by market participants it may be hard for managers to deviate substantially from industry norms. However, analyzing this case is interesting because it highlights the importance of uncertainty about fees on investors' returns.

Firms conjecture the size of the fund Q_t^C in trying to estimate the talent of the manager. Firms update their beliefs as follows:

$$E_{t+1}\left[X|h_t\right] = w_t E_t[X|h_{t-1}] + (1-w_t)\left(X - c(Q_t) + c(Q_t^C)\right) \tag{5}$$

From (5) it is clear that decreasing fund size positively affects firms' estimation of talent. As a result, the manager limits the fund's size. However, the manager's choice of fees does not affect the inference investors make about his ability. In other words, there is nothing to prevent the manager from fully capturing the return he generates for the fund. As the following proposition argues, however, the manager still limits the size of the fund compared to the case of no information asymmetry about size. Therefore, some degree of unobservability of fund fees seems to be critical to derive the performance persistence result even if it is not critical in generating incentives to limit fund size. However, as we discuss below, a possible extension to the model set-up produces positive and persistent performance even when fees are observable and freely determined by the manager.

Proposition 8 *When fees are observed but fund size is unobserved, the manager limits fund size by choosing an amount lower than the amount he would choose if fund size were observed. However, the manager captures all the surplus he generates and investors' expected return* α_t *is zero.*

In our model the only factor that affects the likelihood of a manager matching with high quality firms is his perceived ability. This simple setup allows us to focus our attention to the role of the matching process and information asymmetry in explaining return patterns in private equity funds. It is, however, possible to extend the model to one in which the entrepreneurs also care about the size of the fund. For example, the value added to firms may decrease as fund size decreases as in Fulghieri and Sevilir (2009). If the firms do not observe the final size of the fund at the time of matching, the fund manager may need to reassure firms that the number of firms that it takes on is not going to be excessive. In this case the fund's fees can act as a commitment device for not increasing size and may lead to performance persistence even when fees are observed. We analyze this case next.

2.2.5 Using Fees in Committing to a Small Fund Size

In the previous section we showed that, when the fees charged by the fund to investors are perfectly observable to all, the fund manager restricts the size of the fund as a result of his attempt to manipulate firms' beliefs, but he extracts all surplus from investors through the fee structure. Here, we show that, even if fees are perfectly observable, small variations of the basic model once again yield the result that the fund manager may optimally choose to provide investors with a positive excess expected return.

We introduce one such extension by assuming that the manager may need to commit to maintain a relatively small fund. This would be the case, for instance, if running a large fund leaves a manager relatively little time to dedicate to individual firms, so that their value added is likely to be low when the fund gets too large (Lopez-de-Silanes, Phalippou and Gottschalg, 2009; Cumming and Dai, 2010). If firms have some outside opportunity, such as to match randomly with another provider of funds, or to retain the option of remaining independent in hopes of getting a better offer at some later point, this would imply a maximum fund size for a given level of ability, to which managers would have to credibly commit in order to match with good firms. Specifically, assume that fees are determined first and that fund size Q_t is not observed at the time entrepreneurs decide to accept managers offer. If entrepreneurs believe that fund size will exceed some cutoff size Q^*, they reject the offer. Hence, it is important for the fund manager to reassure entrepreneurs that the fund's size will remain below this threshold.

We only analyze the most interesting case when the size threshold is binding in the sense that, absent other concerns, the fund manager optimally chooses a fund size larger than the threshold. Denoting by Q'_t the solution to the first order condition for maximizing the fund manager's return in the absence of the constraint,

$$\frac{\partial E[V_t]}{\partial Q_t} = v_t P_t \left(W_t - Q_t c'\left(Q_t\right)\right) + f_t \left(1 - v_t\right) + \frac{\partial V_{t+1}}{\partial E_{t+1}[X]} \frac{\partial E_{t+1}[X]}{\partial Q_t} = 0,$$

this means that $Q'_t > Q^*$, at least for values of $v_t \approx 1$ and $f_t \approx P_t W_t$, the values that would be chosen if the manager set fees so as to capture all the surplus.

With these changes to the basic model, we can now state the following result.

Proposition 9 *When the fund manager needs to commit to a maximum fund size Q^*, there exist parameter values such that: (1) the equilibrium scale is $Q_t \leq Q^*$; (2) $v_t < 1$ and $f_t < P_t W_t$; and (3) $\alpha_t > 0$, even when all fees are perfectly and publicly observed.*

The perfect observability of fees makes the manipulation of entrepreneurs' beliefs much more difficult, since entrepreneurs can easily back out the total fund returns from observing the actual fees that are paid to the manager. This proposition demonstrates that the finding of positive and persistent fund returns is robust to the possibility that fees are observed if we assume that fund managers may have other considerations in determining fees. In this case, setting a relatively low fee structure, and one that fails to fully extract all surplus from investors, reduces the incentives of the fund manager to raise too much capital from investors and thus provides a commitment to run a relatively small fund.

As a final point, in the proof of Proposition 9 we also show that the optimality of providing positive excess returns (i.e., $\alpha_t > 0$) relies crucially on the incentive the fund manager has to manipulate entrepreneurs' beliefs concerning his talent. Absent such incentive for the fund manager, the optimal fee structure will always fully extract all surplus from investors, even if the manager needs to commit to maintain a relatively small fund size, as analyzed above. This occurs because, in the absence of an incentive to manipulate firms' beliefs, any fee structure that achieves commitment can be replaced by another structure that achieves the same commitment but that fully extracts all the rents from investors. This, however, cannot always happen when there is an incentive for manipulating beliefs of entrepreneurs.

Put differently, the signal jamming argument used throughout our analysis is a crucial requirement for "leaving money on the table," even when the fund manager uses fee structure as a commitment device to keep fund size small.

3 Empirical Predictions

In addition to explaining why managers may limit fund size and provide persistent positive returns to investors, we provide several novel cross sectional and time series predictions about the performance of private equity funds.

In most instances of venture capital financing, the entrepreneur retains a significant stake in the firm, especially when his input (i.e., effort) is important for the firm. Therefore, entrepreneurs have an incentive to match with fund managers that can add value to their firms. On the other hand, in leveraged buyout (LBO) deals existing investors are paid ex-ante and most of the time sell the bulk of their stake in the company. Therefore, firms that are targets of a buyout are not concerned about inferring the value-adding ability of the manager. As a result, managers of LBO funds should have little incentive to either keep fund size small or charge lower fees to their investors as a way of manipulating the beliefs of entrepreneurs at the target firms. Our model therefore distinguishes between the returns of venture capital versus LBO funds. This prediction, to the best of our knowledge, is unique to our framework, and is consistent with recent empirical findings. For example, Kaplan and Schoar (2005) find that the coefficient of past returns in estimating future returns (performance persistence) is much lower for LBO funds compared to venture capital funds. Consistent with this, Metrick and Yasuda (2008) find that LBO firms scale their size much faster in reaction to past positive returns compared to venture capital firms.

There are also cross sectional differences among venture capital funds in terms of their focus on early versus later stages (Gompers and Lerner, 1999). Empirical evidence finds that VC's value added is more pronounced in the first two rounds of financing (Chemmanur, Krishnan and Nandy, 2009). Therefore, the matching process should be more important for

VC funds that specialize in earlier round investing, with the implication that our findings concerning fund returns and persistence should be more applicable to these funds. To the best of our knowledge, this also is a novel prediction of our framework.

Although we do not explicitly model how competition affects the matching process between firms and funds, a manager's incentive to manipulate the beliefs of firms should depend on the degree of competition among venture capital funds to attract good target firms. With no competition, a manager has no incentive to manipulate beliefs because he will be able to match with good firms regardless. On the other hand, too much competition will decrease the marginal impact of manipulation on the matching probability with good firms. Therefore, we predict that the relationship between competition and performance persistence should be nonlinear, with most of the effect emphasized for intermediate levels of competition. If individual markets exhibit different degrees of competition - either because of geographic separation (Chen, Gompers, Kovner and Lerner, 2009), specialization (Fulghieri and Sevilir, 2009; Gompers, Kovner, Lerner, Scharfstein, 2008), or fund availability over time, one could test whether performance persistence varies within these dimensions. These predictions we believe have not been tested so far.

In the time series, as the manager gets more experienced, firms have better information about the ability of the manager. Over time a fund manager's incentive to manipulate firms' beliefs should decrease. As a result we expect a positive relationship between past fund returns and changes in fund size. This is consistent with recent evidence in Kaplan and Schoar (2005). Our model also predicts that the learning that takes place over time about a fund manager's ability implies larger fees by more experienced managers, as found by Gompers and Lerner (1999).

4 Conclusion

Anecdotal evidence suggests that many successful private equity funds are oversubscribed. On the other hand, private equity funds appear to generate persistent abnormal returns for their investors, in contrast to mutual funds, which exhibit little or no performance persistence. We argue that private equity funds are fundamentally different from mutual funds because of two reasons: First, private equity funds need to match with good firms, which want to match with managers who have higher ability to add value. Second, there is greater asymmetry of information regarding managers ability to add value because many fund characteristics are not fully observed by firms. Therefore, there is a high incentive for private equity fund managers to attempt to manipulate the beliefs of firms about their ability. In particular, by charging lower fees and/or limiting the fund's size, a manager tries to improve firms' beliefs about his ability to add value. In the signal jamming equilibrium we develop, firms are not fooled and they correctly form an unbiased expectation. Nevertheless, the managers limits the fund's size and/or fees and provides persistent returns because otherwise his probability of matching with good firms decreases. Our model not only explains differences in performance persistence between mutual and private equity funds but also discusses how our results would vary across different types of funds, fund's stage focus, managers' experience, competition across markets and over time.

Appendix

Proof of Proposition 1: The first order conditions to the fund manager's maximization problem are

$$\frac{\partial E[V_t]}{\partial Q_t} = v_t(P_tW_t - f_t) - Q_tv_tP_tc'(Q_t) + f_t - \lambda_t(1 - v_t)P_tc'(Q_t) = 0, \tag{6}$$

$$\frac{\partial E[V_t]}{\partial f_t} = (Q_t - \lambda)(1 - v_t) = 0, \tag{7}$$

$$\frac{\partial E[V_t]}{\partial v_t} = (Q_t - \lambda)(P_tW_t - f_t) = 0, \tag{8}$$

$$\lambda_t(1 - v_t)(P_tW_t - f_t) = 0. \tag{9}$$

There are a number of possibilities. Suppose that $\lambda_t = 0$. Then (7) can only be satisfied by $v_t = 1$. Also, (8) can only be satisfied if $P_tW_t - f_t = 0$. Alternatively assume alpha is positive but not equal to Q. Again (7) can only be satisfied by $v_t = 1$. Also, (8) can only be satisfied if $P_tW_t - f_t = 0$. Finally assume alpha is equal to Q_t, in this case either $v_t = 1$ or $P_tW_t - f_t = 0$. In all these possible cases the solution for Q_t from (6) is determined from :

$$\frac{\partial E[V_t]}{\partial Q_t} = -Q_tc'(Q_t) + X_t - c(Q_t) = 0 \tag{10}$$

This completes the proof. □

Proof of Proposition 2: Suppose that $\lambda_t = 0$. Then (7) can only be satisfied by $v_t = 1$. Also, (8) can only be satisfied if $P_tW_t - f_t = 0$, or in other words if $f_t = P_tW_t$. Note that in equilibrium we have $\alpha_t = 0$ even under the assumption that $\lambda_t = 0$. Suppose instead that $\lambda_t > 0$. Then from (9) either $v_t = 1$, $f_t = P_tW_t$, or both. From (8) we also know that either $Q_t = \lambda_t$, or $f_t = P_tW_t$. Note that either way, $\alpha_t = 0$. □

Proof of Proposition 3: The Kuhn-Tucker conditions are

$$v_t(P_tW_t - f_t) - Q_tv_tP_tc'(Q_t) + f_t - \lambda_tP_tc'(Q_t)(1 - v_t) + C(e_t) = 0 \tag{11}$$

$$\frac{\partial E[V_t]}{\partial f_t} = (Q_t - \lambda)(1 - v_t) = 0, \tag{12}$$

$$\frac{\partial E[V_t]}{\partial v_t} = (Q_t - \lambda)(P_t W_t - f_t) = 0, \tag{13}$$

$$\lambda_t(1 - v_t)(P_t W_t - f_t) = 0 \tag{14}$$

$$Q_t v_t P_t - Q_t \frac{\partial C(e_t)}{\partial e_t} + \frac{\partial V_{t+1}}{\partial e_t} \tag{15}$$

Suppose that $\lambda_t = 0$. Then (12) can only be satisfied by $v_t = 1$. Also, (13) can only be satisfied if $P_t W_t - f_t = 0$, or in other words if $f_t = P_t W_t$. Note that in equilibrium we have $\alpha_t = 0$ even under the assumption that $\lambda_t = 0$. Suppose instead that $\lambda_t > 0$. Then either $v_t = 1$, $f_t = P_t W_t$, or both. From (8) we also know that either $Q_t = \lambda_t$, or $f_t = P_t W_t$. Note that either way, $\alpha_t = 0$. □

Proof of Proposition 4 The Kuhn-Tucker conditions are given by (1), (2), and (3). When the manager's effort is not observable we have $\frac{\partial V_{t+1}}{\partial e_t} > 0$ in (3), which increases the equilibrium level of effort compared to the case when effort is not observable.

The effect of effort on the manager's payoff when effort is observable is equal to:

$$Q_t P_t e_t - Q_t C(e_t) \tag{16}$$

Define the optimal level of effort when effort is observable as e_t^{FB}, which basically maximizes $P_t e_t - C(e_t)$. Given that the manager exerts higher effort than what is socially optimal, the inequality following must hold: $P_t e_t - C(e_t) < P_t e_t^{FB} - C(e_t^{FB})$. Close examination of the Kuhn-Tucker condition with respect to Q_t reveals that effect of effort on the optimization equation of Q_t is limited to the term $P_t e_t - C(e_t)$. Given that this term is smaller than $P_t e_t^{FB} - C(e_t^{FB})$, the equilibrium level of quantity will be smaller than when effort is observable and the quantity is socially optimal. □

Proof of Proposition 5: From the Bellman equation provided in the text, we obtain the Kuhn-Tucker conditions, which are

$$\frac{\partial E[V_t]}{\partial Q_t} = v_t(P_tW_t - f_t) - Qv_tP_tc'(Q_t) + f_t - \lambda_tP_tc'(Q_t)(1 - v_t) = 0 \tag{17}$$

$$\frac{\partial E[V_t]}{\partial f_t} = (Q_t - \lambda_t)(1 - v_t) + \frac{\partial V_{t+1}}{\partial E_{t+1}[X]}\frac{\partial E_{t+1}[X]}{\partial f_t} = 0 \tag{18}$$

$$\frac{\partial E[V_t]}{\partial v_t} = (Q_t - \lambda_t)(P_tW_t - f_t) + \frac{\partial V_{t+1}}{\partial E_{t+1}[X]}\frac{\partial E_{t+1}[X]}{\partial v_t} = 0 \tag{19}$$

$$\lambda_t(1 - v_t)(P_tW_t - f_t) = 0 \tag{20}$$

Assume that $\lambda_t > 0$, so that the investors' participation constraint binds. In this case, either $1 - v_t = 0$ or $P_tW_t - f_t = 0$ or both. However this can't be true because we could not then satisfy equations (18) and (19). Therefore, λ_t must be equal to zero. In this case again $1 - v_t$ or $P_tW_t - f_t$ cannot be equal to zero. As a result investors' expected return α_t is positive. We can find the solutions for f_t and v_t from (18) and (19) and plug this into (17) to solve for the managers choice of Q_t. The manager accepts a lower quantity than investors are willing to invest because the fund's return is positive. □

Proof of Proposition 6: We can write the Bellman equation as

$$V_t(E_t[X|h_{t-1}]) = \max_{Q_t, f_t, v_t} [Q_tv_t(P_tW_t - f_t) + Q_tf_t + V_{t+1}(E_{t+1}[X|h_t])]$$

subject to the constraints,

$$(1 - v_t)(P_tW_t - f_t) \geq 0$$

$$E_{t+1}[X|h_t] = w_tE_t[X|h_{t-1}] + (1 - w_t)\left(\frac{\alpha_t}{(1 - v)P_t} + \frac{f_t}{P_t} + c\left(Q_t^C\right)\right)$$

The Kuhn-Tucker conditions are

$$\frac{\partial E[V_t]}{\partial Q_t} = v_t(P_tW_t - f_t) - Q_tv_tP_tc'(Q_t) + f_t - \lambda_tP_tc'(Q_t)(1-v_t) + \frac{\partial V_{t+1}}{\partial E_{t+1}[X]}\frac{\partial E_{t+1}[X]}{\partial Q_t} = 0 \quad (21)$$

$$\lambda_t(1-v_t)(P_tW_t - f_t) = 0 \quad (22)$$

Let's assume that $\lambda_t > 0$ in other words the investors participation constraint binds or $\alpha_t = 0$. In this case $P_tW_t - f_t = 0$ given that $v_t < 1$. If $P_tW_t - f_t = 0$ equation 21 cannot be satisfied if $-\frac{\partial V_{t+1}}{\partial E_{t+1}[X]}\frac{\partial E_{t+1}[X]}{\partial Q_t} > f_t$. If $\lambda_t = 0$ the same condition also ensures that $P_tW_t - f_t > 0$. □

Proof of Proposition 7: Kuhn-Tucker conditions are

$$\frac{\partial E[V_t]}{\partial Q_t} = v_t(P_tW_t - f_t) - Q_tv_tP_tc'(Q_t) + f_t - \lambda_tP_tc'(Q_t)(1-v_t) + \frac{\partial V_{t+1}}{\partial E_{t+1}[X]}\frac{\partial E_{t+1}[X]}{\partial Q_t} = 0 \quad (23)$$

$$\frac{\partial E[V_t]}{\partial f_t} = (Q_t - \lambda_t)(1 - v_t) + \frac{\partial V_{t+1}}{\partial E_{t+1}[X]}\frac{\partial E_{t+1}[X]}{\partial f_t} = 0 \quad (24)$$

$$\frac{\partial E[V_t]}{\partial v_t} = (Q_t - \lambda_t)(P_tW_t - f_t) + \frac{\partial V_{t+1}}{\partial E_{t+1}[X]}\frac{\partial E_{t+1}[X]}{\partial v_t} = 0 \quad (25)$$

$$\lambda_t(1-v_t)(P_tW_t - f_t) = 0 \quad (26)$$

Let's assume that $\lambda_t > 0$ in other words the investors participation constraint binds. In this case there are three possibilities either $1 - v_t = 0$ or $P_tW_t - f_t = 0$ or both. If $P_tW_t - f_t = 0$, then (25) cannot be satisfied and if $1 - v_t = 0$ then (24) cannot be satisfied. As a result λ_t must be zero. When $\lambda_t = 0$ both $1 - v_t > 0$ and $P_tW_t - f_t > 0$ otherwise (24) and (25) cannot be satisfied, respectively.

For all possible cases discussed, (23) has an additional negative term compared to (6), which implies that the optimal quantity that satisfies (23) is lower than the Q_t^*. □

Proof of Proposition 8: The Bellman equation is similar to previous cases but this time f_t and v_t is observable therefore managers choice of fee structure does not affect firms expectation about the talent of the manager.

$$V_t(E_t[X|h_{t-1}]) = \max_{Q_t, f_t, v_t} \left[Q_t v_t(P_t W_t - f_t) + Q_t f_t + V_{t+1}(E_{t+1}[X|h_t])\right]$$

subject to the constraints,

$$(1 - v_t)(P_t W_t - f_t) \geq 0$$

$$E_{t+1}[X|h_t] = w_t E_t[X|h_{t-1}] + (1 - w_t)\left(\frac{\alpha_t}{(1 - v_t)P_t} + \frac{f_t}{P_t} + c\left(Q_t^C\right)\right)$$

The Kuhn-Tucker conditions are as follows:

$$\frac{\partial E[V_t]}{\partial Q_t} = v_t(P_t W_t - f_t) - Q_t v_t P_t c'(Q_t) + f_t - \lambda_t P_t c'(Q_t)(1 - v_t) + \frac{\partial V_{t+1}}{\partial E_{t+1}[X]} \frac{\partial E_{t+1}[X]}{\partial Q_t} = 0 \quad (27)$$

$$\frac{\partial E[V_t]}{\partial f_t} = (Q_t - \lambda_t)(1 - v_t) = 0 \quad (28)$$

$$\frac{\partial E[V_t]}{\partial v_t} = (Q_t - \lambda_t)(P_t W_t - f_t) = 0 \quad (29)$$

$$\lambda_t(1 - v_t)(P_t W_t - f_t) = 0 \quad (30)$$

Suppose that $\lambda_t = 0$. Then (28) can only be satisfied by $v_t = 1$. Also, (29) can only be satisfied if $P_t W_t - f_t = 0$, or in other words if $f_t = P_t W_t$. Note that in equilibrium we have $\alpha_t = 0$ even under the assumption that $\lambda_t = 0$. Suppose instead that $\lambda_t > 0$ then $(1 - v_t)(P_t W_t - f_t) = 0$ in other words $\alpha_t = 0$.In all possible cases equation (27) simplifies to:

$$\frac{\partial E[V_t]}{\partial Q_t} = -Q_t c'(Q_t) + X_t - c(Q_t) + \frac{\partial V_{t+1}}{\partial E_{t+1}[X]} \frac{\partial E_{t+1}[X]}{\partial Q_t} = 0 \quad (31)$$

It is obvious from this equation that managers choice of quantity will be lower than the optimal quantity because the additional term $\frac{\partial V_{t+1}}{\partial E_{t+1}[X]} \frac{\partial E_{t+1}[X]}{\partial Q_t}$ compared to (10) is negative. □

Proof of Proposition 9: Assuming that $\frac{\partial^2 E[V_t]}{\partial Q_t^2} < 0$, which is a necessary condition for the fund manager's maximization problem to be well defined, we can invoke the Implicit

Function Theorem to determine that $\frac{\partial Q_t'}{\partial v_t}$ is proportional to

$$\frac{\partial^2 E[V_t]}{\partial v_t \partial Q_t} = P_t \left(W_t - Q_t c'\left(Q_t\right)\right) - f_t = \left(P_t W_t - f_t\right) - P_t Q_t c'\left(Q_t\right).$$

For $f_t \approx P_t W_t$, this reduces to $\frac{\partial^2 E[V_t]}{\partial v_t \partial Q_t} = -P_t Q_t c'\left(Q_t\right) < 0$ for $Q_t > 0$. By contrast, $\frac{\partial Q_t'}{\partial f_t}$ is proportional to

$$\frac{\partial^2 E[V_t]}{\partial f_t \partial Q_t} = \left(1 - v_t\right),$$

which is weakly positive, and strictly positive for $v_t < 1$.

Since by assumption $Q_t' > Q^*$ for $v_t \approx 1$ and $f_t \approx P_t W_t$, the only way to commit to choose $Q_t \leq Q^*$ is by reducing f_t, since reducing v_t would actually increase Q_t. The impact of such reductions, however, is minimal for $v_t = 1 - \epsilon$. Note, however, that as f_t becomes smaller, $\frac{\partial^2 E[V_t]}{\partial v_t \partial Q_t} = \left(P_t W_t - f_t\right) - P_t Q_t c'\left(Q_t\right)$ may be either positive or negative, and will strictly positive for small enough Q_t. Such Q_t will be optimal when the signal jamming incentive is sufficiently strong: $\frac{\partial V_{t+1}}{\partial E_{t+1}[X]} \frac{\partial E_{t+1}[X]}{\partial Q_t} << 0$. This can be seen by noting that if the incentive to signal jam is sufficiently large in absolute magnitude, either because $\frac{\partial V_{t+1}}{\partial E_{t+1}[X]} >> 0$ or $\frac{\partial E_{t+1}[X]}{\partial Q_t} << 0$, or both, then the first order condition can only be satisfied with $W_t - Q_t c'\left(Q_t\right) > 0$. From this, it is now straightforward to see that the only way to credibly commit to choose $Q_t \leq Q^*$ is by choosing some $v_t < 1$ and some $f_t < P_t W_t$, as desired.

We also establish here the necessity of a signal jamming incentive for this result. Suppose to the contrary, and that the term $\frac{\partial V_{t+1}}{\partial E_{t+1}[X]} \frac{\partial E_{t+1}[X]}{\partial Q_t} = 0$, so that there is either no incentive or no ability to influence entrepreneurs' belifes. In this case, the fund manager's first order condition for the case where $0 < v_t < 1$ and $0 < f_t < P_t W_t$ reduces to

$$\frac{\partial E[V_t]}{\partial Q_t} = v_t P_t \left(W_t - Q_t c'\left(Q_t\right)\right) + f_t \left(1 - v_t\right) = 0. \tag{32}$$

For any $v_t < 1$, note that this FOC can only be satisfied if $W_t - Q_t c'\left(Q_t\right) < 0$. Therefore, it is straightforward to see from this FOC that $\frac{\partial Q_t}{\partial v_t} < 0$ and $\frac{\partial Q_t}{\partial f_t} > 0$.

Suppose now that there is some combination $0 < v_t < 1$ and $0 < f_t < P_t W_t$ that deliver $Q_t = Q^*$. Since $\frac{\partial Q_t}{\partial v_t} < 0$ and $\frac{\partial Q_t}{\partial t_t} > 0$, the fund manager should be able to increase v_t by some ϵ and at the same time increase f_t by some δ such that Q_t remains constant at Q^*, but now the manager makes higher profits since both fees have been increased. Adjusting the fees in this way will always be possible until either $v_t = 1$ or $f_t = P_t W_t$. Therefore, any fee structure that achieves commitment can be replaced by another structure that achieves the same commitment but that fully extracts all the rents from investors. We can now conclude that absent the signal jamming incentive it is never optimal to leave money on the table. $\square$

References

Bao, Jack and Alex Edmans, 2009, "Do Investment Banks Have Skill? Performance Persistence of M&A Advisors," Working Paper.

Baum, J. A., and B. S. Silverman, 2004, "Picking Winners or Building Them? Alliance, Intellectual, and Human Capital as Selection Criteria in Venture Financing and Performance of Biotechnology Startups," *Journal of Business Venturing*, 19, 411–436.

Berk, J., and R. Green, 2004, "Mutual Fund Flows and Performance in Rational Markets," *Journal of Political Economy* 112, 1269-1295.

Busse, Jeff, Amit Goyal and Sunil Wahal, 2009, "Performance Persistence in Institutional Investment Management," forthcoming in *Journal of Finance*.

Carhart, Mark M., 1997, "On Persistence in Mutual Fund Performance," *Journal of Finance*, Vol. 52 No. 1.

Chen, Henry, Paul Gompers and Anna Kovner and Josh Lerner, 2009, "Buy Local? The Geography of Successful and Unsuccessful Venture Capital Expansion," *NBER Working Papers 15102*.

Chemmanur, Thomas J and Paolo Fulghieri, 1994, "Reputation, Renegotiation, and the Choice between Bank Loans and Publicly Traded Debt," *Review of Financial Studies*, vol. 7(3), pages 475-506.

Chemmanur, Thomas J., Karthik Krishnan and Debarshi Nandy, 2009, "How Does Venture Capital Financing Improve Efficiency in Private Firms? A Look Beneath the Surface," Working paper.

Cochrane, J., 2005, "The Risk and Return of Venture Capital", *Journal of Financial Economics* 75, 3-52.

Cumming, Douglas J. and Na Dai, 2010, "Fund Size, Limited Attention and Valuation of Venture Capital Backed Firms " *Working paper.*

Cumming, Douglas J. and Uwe Walz, 2009, "Private Equity Returns and Disclosure Around the World," forthcoming at *Journal of International Business Studies*.

Fernando, Chitru S., Vladimir A. Gatchev and Paul A. Spindt, 2005, "Wanna Dance? How Firms and Underwriters Choose Each Other," *Journal of Finance*, Vol. 60, pp. 2437-2469.

Fulghieri, Paolo and Merih Sevilir, 2009, "Size and Focus of a Venture Capitalist's Portfolio," forthcoming at *Review of Financial Studies*.

Glode, V., and R. Green, 2008, "Information Spill-Overs and Performance Persistence in Private Equity Partnerships," Unpublished working paper, Carnegie-Mellon University.

Gompers, P.A., and J. Lerner, 1999, "An Analysis of Compensation in the U.S. Venture Capital Partnership," *Journal of Financial Economics* 51, 3-44.

Gompers, P.A., and J. Lerner, 2001, The Money of Invention: How Venture Capital Creates New Wealth, Harvard Business School Publishing, Boston.

Gompers, P. A., Anna Kovner and Josh Lerner, 2009, "Specialization and Success: Evidence from Venture Capital," *Journal of Economics & Management Strategy*, Vol. 18, Issue 3, pp. 817-844.

Gorman, M., and W. A. Sahlman, 1989, "What Do Venture Capitalists Do?," *Journal of Business Venturing*, 4, 231–248.

Hellmann, T., and M. Puri, 2000, "The Interaction between Product Market and Financing Strategy: The Role of Venture Capital," *Review of Financial Studies*, 13, NO. 4, 959–984.

Hellmann, T., and M. Puri, 2002, "Venture Capital and the Professionalization of Start-Up Firms: Empirical Evidence," *The Journal of Finance*, LVII, NO. 1, 169–197.

Hochberg, Y., A. Ljungqvist, A. Vissing-Jorgensen, 2008, "Informational Hold-up and Performance Persistence in Venture Capital," working paper.

Hochberg, Y.V., A. Ljungqvist, and Y. Lu, 2007, "Whom You Know Matters: Venture Capital Networks and Investment Performance," *Journal of Finance* 62, 251-301.

Holmstrom, B., 1999, "Managerial Incentive Problems: A Dynamic Perspective," *Review of Economic Studies*, 66, 169-182.

Hsu, D. H., 2004, "What Do Entrepreneurs Pay for Venture Capital Affiliation?," *The Journal of Finance*, LIX, NO. 4, 1805–1844.

Inderst, R., H. Mueller, and F. Muennich, 2007, "Financing a Portfolio of Projects," *Review of Financial Studies* 20: 1289-1325.

Jones, C., and M. Rhodes-Kropf, 2003, "The Price of Diversifiable Risk in Venture Capital and Private Equity," Unpublished working paper, Columbia University.

Kaplan, S.N., and A. Schoar, 2005, "Private Equity Performance: Returns, Persistence and Capital Flows," *Journal of Finance* 60, 1791-1823.

Kaplan, S.N., B. Sensoy, and P. Stromberg, 2002, "How Well Do Venture Capital Databases Reflect Actual Investments?", Unpublished working paper, University of Chicago.

Korteweg, A., and M. Sorensen, 2008, "Estimating Risk and Return of Infrequently Traded Assets: A Bayesian Selection Model of Venture Capital," Unpublished working paper, Columbia University.

Lerner, J., and A. Schoar, 2004, "The Illiquidity Puzzle: Evidence from Private Equity Partner- ships," *Journal of Financial Economics* 72, 3-40.

Litvak, K., 2009, "Venture Capital Limited Partnership Agreements: Understanding Compensation Arrangements," forthcoming at *University of Chicago Law Review*.

Ljungqvist, A. and M. P. Richardson, 2003, "The Cash Flow, Return and Risk Characteristics of Private Equity," NYU, Finance Working Paper No. 03-001.

Lopez-de-Silanes, F. and L. Phalippou and O. Gottschalg, 2009, "Giants at the Gate: Diseconomies of Scale in Private Equity" working paper.

Megginson, W. L., and K. A. Weiss, 1991, "Venture Capitalist Certification in Initial Public Offerings," *The Journal of Finance*, XLVI, NO. 3, 879–903.

Metrick, A. and Yasuda, A., 2009, "The Economics of Private Equity Funds," forthcoming at *Review of Financial Studies*.

Phalippou, L. and O. Gottschalg, 2008, "The Performance of Private Equity Funds," forthcoming at *Review of Financial Studies*.

Phalippou, Ludovic, 2009, “Venture Capital Funds: Performance Persistence and Flow-Performance Relation,” forthcoming *Journal of Banking and Finance*.

Quigley, John M. and Susan E. Woodward, 2003, “An Index for Venture Capital,” Department of Economics, UCB working paper.

Sorensen, Morten, 2007, “How Smart is Smart Money: A Two-Sided Matching Model of Venture Capital,” *Journal of Finance*, 62, 2725-2762

Stein, J., 1989, “Efficient Capital Markets, Inefficient Firms: A Model of Myopic Corporate Behavior,” *Quarterly Journal of Economics*, 104, 655-669.

Titman, Sheridan and Brett Trueman, 1986, “Information quality and the valuation of new issues,” *Journal of Accounting and Economics*, vol. 8(2), pages 159-172.